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THE PRODUCTION OF AXIALLY SYMMETRIC MAGNETIC FIELDS

and

NEW POWER SOURCES FOR LARGE BETATRONS AND SYNCHROTONS

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R.L. Garwin

ABSTRACT

In the first half of the paper a method for producing axially symmetric magnetic fields of arbitrary form by means of air core coils wound on axially symmetric surfaces of arbitrary extent and shape is set forth. Applications of the principle to betatrons and mass spectrometers, etc., are noted, the calculations being carried out in detail for an air core betatron. As a byproduct of the investigation a simple system for obtaining rigorously $\sum_{m} \varepsilon^2$ is described, and proof is given that this sum is a criterion for adjusting variables toward balance in many types of computers. A computer designed especially for coil design was built and used with excellent results, but because of the pressure of time no construction data for actual coils are given.

The second part of the thesis is devoted to an exposition of two new types of energy storage devices # as applied especially to betatrons and synchrotrons. Both these devices employ kinetic energy as a very much more powerful means of energy storage than either electrostatic or magnetic. The first machine consists of a flywheel operated as a Faraday disc with a series field coil in order to get high voltages and power. The machine is air core except for the steel disc and is intended for use in the largest betatrons and synchrotrons at repetition rates less than one per second. This device will permit the attainment of electron energies considerably above 5 Bev with the added possibility of

Invented by the author, ~~who is the sole owner of these machines for whatever purpose used. Every person is cautioned to treat this unpublished thesis as a confidential disclosure~~.

accelerating protons and deuterons to approximately the same energy. A small (140 Mev) machine was built (pictures enclosed) and tested with the results expected. The second type of generator consists of a large scale variometer, the circuit electromechanical equations of which are such as to make this machine ideal for the replacement of the usual resonant condenser at great savings in power (more than 85%) and cost (savings better than 95%).

Together with the coil design scheme these two new generators should extend the power and scope of betatron and synshrotron acceleration by factors of 50 with consequent increase in the availability of these accelerators for research work.

THE PRODUCTION OF AXIALLY SYMMETRIC MAGNETIC FIELDS

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INTRODUCTION

There are many cases in which a magnetic field of a certain form is desired; for instance in a betatron the field must be normal to a plane and may be represented as of magnitude $H = H_0\left(\frac{\rho}{R}\right)^{-n}$ over a certain range of radius ρ. For beta-ray spectrometers also, a field of shape determined by focussing conditions should obtain. Cloud chambers, magnetrons, B-H meters all require uniform magnetic fields. It is the purpose of this report to present a method (believed new) by which one may calculate and construct coils to produce axially symmetric fields of otherwise arbitrary characteristics.

In the analogous electrostatic problem it is an eacy matter, given the distribution of charge, to calculate the electric field. It is far more difficult (and usually impossible) to compute the field if boundary values are given as equipotentials; but this problem is easily solved by means of a plotting tank. On the other hand, if one starts from a field configuration dictated by lens properties, etc., one need only apply proper potentials to conductors coinciding with the desired equipotentials to produce the required field. For magnetic fields produced by the magnetization in iron, one need (as an excellent approximation for fields less than 10,000 gauss) simply to treat the iron surface as an equipotential for magnetic

1 A portion of the thesis for B.S. degree.

field and make the pole pieces coincide with an equipotential of the desired field plot. Magnetic fields greater than about 20,000 gauss, however, must be produced without the aid of iron, thus throwing out the equipotential method obtaining pole shapes as a means of shaping coils. (In addition, it is desirable from the standpoints of economy and compactness to eliminate iron from all applications where the air gap is greater than about 1/10 the length of flux path). Here again, given the coil shape, it is a simple matter (but laborious) to calculate the resulting field -- the converse process involves an integral equation of the first type and is peculiarly difficult.

FORMULATION OF PROBLEM

Let us, however, attack this last problem. As the immediate application is to be to an air-core betatron we shall restrict the coil shape slightly. The magnetic field in the region of the orbit is defined by the following equations holding in the plane of the orbit and by harmonic continuation in the neighborhood of this plane

$$H_z = H_0\left(\frac{\rho}{\rho_0}\right)^{-n} \text{ for } |\rho-\rho_0| < \alpha\rho_0 \qquad 0<n<1$$

$$\int_0^{\rho_0} 2\pi H\rho\, d\rho = 2\pi\rho_0^2 H_0$$

$$H_r = H_\theta = 0$$

We shall consider the coils to be plane circular pancakes located at distance b from the plane of the orbit and to have maximum radius $(1+\alpha)\rho_0$.

The direct method of solution appears to involve the expansion in Legendre polynomials of the field due to a circular coil. Unfortunately this method becomes extremely complicated and is so difficult as to be of little use in practice. We shall pursue the following plan. Define the vector $A = \frac{1}{4\pi}\oint \frac{i\,dl}{r}$, then it can be shown that $H = \operatorname{curl} A$ where H is the magnetic field and A the so-called vector potential.

i is ofcourse the current in element dl, and r is the distance from dl to the point under consideration. The integration extends around the circuit. Since we are concerned only with the field in the median plane, and since the coils are symmetrical $H_r = H_\theta = 0$, and by the expansion of the curl in cylindrical coordinates

$$H_z = \frac{\partial A_\theta}{\partial r} + \frac{A_\theta}{r} \Big|_{(r=\rho)} \qquad (1)$$

For two thin circular coils distant b from the median plane with radius variable a, current desity $j(a)$, and radial incremental width da the vector potential in the median plane is given by

$$A = \frac{2}{4\pi}\int_0^{2\pi}\int_0^{\infty} \frac{j(a)\,a\,da\cos\varphi\,d\varphi}{r}$$

$$= \frac{1}{2\pi}\int_{a=0}^{\infty}\int_0^{2\pi} \frac{j(a)\,a\cos\varphi\,d\varphi\,da}{(a^2+b^2+\rho^2-2a\rho\cos\varphi)^{1/2}}$$

and

$$H(\rho) = \frac{1}{\pi\rho}\int_{a=0}^{\infty}\int_{x=-1}^{1} \frac{j(a)\,a\,x\,(a^2+b^2-a\rho x)\,da\,dx}{(1-x^2)^{1/2}\,(a^2+b^2+\rho^2-2a\rho x)^{3/2}} \qquad (2)$$

$$H(\rho) = \int_{a=0}^{a=\infty} j(a)\,g(a,\rho)\,da \qquad \text{if } b = b(a) \qquad (3)$$

It can be shown that (3) has a continuous solution $j(a)$ for $g(a,\rho)$ continuous in the region and for $\frac{\partial g}{\partial x}$ finite. Thus we have proved that there exists a continuous distribution of current $j(a)$ over the two planes distant b from the median such that the field in the median may be of any specified form. This same result may easily be proved for coils wound on the circumference of a cylinder (in this case the required field must be specified along the axis).

2 M. Bocher, Introduction to Study of Integral Equations, Cambridge University Press 1914, p 61.

The analytical solution of (2) is an almost impossible job. Let us see what simplification we can introduce by our knowledge of a specific problem -- the air-core ebtatron. Let us restrict a to the range $0<a<(1+\alpha)\rho_0$ coils could be calculated for $a<\rho_0$ or for that matter for $a<.1\rho_0$ but would be more difficult to build. Then (2) becomes

$$H(\rho) = \int_{a=0}^{(1+\alpha)\rho_0} j(a)\, g(a,\rho)\, da \qquad 0<\rho<(1+\alpha)\rho_0$$

This integral equation may now be approximated by the following set of simultaneous linear agebraic equations

$$H(\rho_1) = \sum_{n=1}^{n=m} j(a_n)\, \Delta a_n\, g(a_n,\rho_1)$$

$$\vdots$$

$$H(\rho_m) = \sum_{n=1}^{n=m} j(a_n)\, \Delta a_n\, g(a_n,\rho_m) \qquad (4)$$

or if the increments are equal

$$[g]\{j\} = \{H\} \qquad (5)$$

$$\text{where } [g] = \begin{vmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & & & \\ g_{n1} & \cdots & \cdots & g_{nn} \end{vmatrix}.$$

We may put these equations in a form more suitable for computation if we write $[L]\{j\} = \{M\}$ where M is the desired mutual inductance of single turn coils placed at radii ρ_n. The M are calculated from the desired field by the relation $M_n = \int_0^{\rho_n} 2\pi\rho H\, d\rho$. The matrix $[L]$ then is symmetric, being the array of numbers L_{mn} giving the mutual inductance between single turn coils of radius a_n and single turn coils of radius ρ_n in planes distant b.

This equation may now be used to calculate arithmetically the necessary current density $j(a)$. To do this we employ the formula[3] $L_{mn} = 8\pi\sqrt{\frac{a_m\rho_n}{c_1}}\left[F(c_1) - E(c_1)\right]$ (6)

3. J.C.Maxwell, Elec. and Mag., first edition, p 341

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where $c_1 = \dfrac{1 - r_2/r_1}{1 + r_2/r_1}$; $\quad r_1^2 = (a+\rho)^2 + b^2$
$\quad r_2^2 = (a-\rho)^2 + b^2$

and F, E are complete elliptic integrals of the first and second kind, respectively. For small c_1, $[F-E]$ is easily calculated as

$$[F-E] = \frac{\pi}{2}\left[\frac{c_1^2}{2} + \frac{3}{8}c_1^4 \cdots\right]$$

It should be noted that the matrix $[\lambda]$ is a function only of b, m and the upper limits of a and ρ. Thus we may use the same inverse, $[\lambda]^{-1}$, to compute $j(a)$ for any desired field $H(\rho)$. The computation of inverse matrices is very painful (and completely impractical for $m > \sim 11$) so that another method would be useful to determine $j'(a)$. Machines for solving large numbers of simultaneous equations are very expensive, electrical ones containing of the order of $2m(m+2)$ potentiometers. The setup to be outlined uses only $4m$ cheap pots.

Two leads are brought out from each of m thin pickup coils having the same number of turns and radii ρ_m . These coils are all wound in one plane. A similar set of m inducing coils is wound with radii a_n . The two planes are then separated a distance b (see fig2) One end of each pickup coil is connected to the movable contact on a potentiometer (two potentiometers, actually, are used for each coil-- one having a total resistance about 100 times that of the other; thus by setting first the coarse potentiometer and the the fine one an accuracy of setting of 1 part in 20,000 may be conveniently and stably obtained). All these potentiometers have their resistance elements connected across two busses supplied by a phase shifting network from the voltage supply which powers the inducing coils 60 cps ac. The potentiometers are set by comparison with two standard resistance boxes so that the voltages from the fingers to ground are in each case proportional to the desired voltage induced in the corresponding pickup coil (without attention to algebraic sign). The sign of the desired induced voltages is taken care of by reversing switches on the pickup coils so that in all cases the voltage from the free end of each pickup coil to ground is the difference between the desired voltage and the actual voltage being generated in the pickup coil. see Fig.1. The currents in the inducing coils are controlled by simple power rheostats of resistance many times the impedance of the inducing coils. This, of course, reduces phase difficulties, but does not entirely eliminate them. No phase shifts are introduced in the pickup coil networks (the bucking potentiometers may be set either with dc and a galvanometer (micro-ammeter) or with ac and an amplifier, with the same setting within .01%; the detecting device attached to the free end of the pickup coils has sufficiently high impedance so that the inductance of the pickup coils produce no phase shift in the metered voltage). The induced voltages, however, vary up to several degrees in phase from exact quadrature with the voltage supplied to the inducing current rheostats. Thus at exact balance for the quadrature components there will still be appreciable voltages from the pickup coils to ground. This difficulty was overcome by the construction of a phase-sensitive amplifier (for one type of balancing scheme). This scheme was the usual Gauss-Seidel or classical iterative method usually employed in simultaneous equation solvers whereby a balance (a solution) is usually obtained by adjusting the independent variable cyclicly so that each equation in turn is satisfied. This method is usually tedious and sometimes fails to converge entirely, but often yields a solution in perhaps 15 minutes for 10 variables. Another disadvantage of the method becomes evident if one attempts to get one of a certain family of solutions with a special property (maximum efficiency, perhaps) by using more variables than points at which the field is specified. In such a case it would be advisable to have an overall picture of the closeness of approximation to the solution at any time.

It has been suggested by several writers that the sum of the absolute magnitudes of the error voltages would provide a balancing criterion, i.e., would behave in such a manner that by minimizing this quantity by varying each potentiometer in turn one would ultimately reduce it to zero. An interesting byproduct of this investigation is the discovery that neither this scheme nor the one employing the peak value of the error signals works at all. The reason is that relative minima are obtained; rotation of any rheostat in either direction increases each of these quantities for certain values of the variables. This is easily seen mathematically and will not be discussed further. However the sum of the squares of the error signal behaves in the desired manner, having no minima other than the solution of the equations. Since the sum of the squares of the error signals is a continuous function of the equation variables the only way in which a monotone decreasing (with successive adjustments) set of values for this function can fail to be obtained is to have a relative minimum -- to have the partial derivatives of this sum with respect to each rheostat variable be zero. Setting up these equations and applying the well-known principle that the determinant of the coefficients of the variables in a set of homogeneous linear simultaneous equations must be zero if the set is to have solutions not identically zero we find that the criterion for a relative minimum is $\|h\|=0$. However, for a solution of the original equations to exist at all, and in general, this determinant must be different from zero. Thus we have shown that the sum of the squares of the errors is monotone decreasing with cyclic adjustment of the variable rheostats and since the function is always positive it must have limit zero -- i.e., the method converges.

The hitch in the application of this method to the setup we have here is the difficulty of devising a mechanism which will neglect out-of-phase components of the error signals. However this has been accomplished by using a synchronous distributor, wide-band amplifier, and phase-discriminating mechanical modulator, together with a thermal meter. The m contact points of the distributor are connected each to the free end of one pickup coil, while the rotating finger goes to the amplifier (all rotating parts are powered by a 3600 rpm motor running synchronous with the power supplied to the calculator). The distributor is driven through a $m:1$ reduction gear from the motor shaft, the only requirement on the distributor contacts is that the finger touch each for more than one half-cycle of the ac power. The signals are then amplified by factor of 10^5 with the output stage of the amplifier a pentode. This pentode feeds a condenser (whose impedance is low compared to the plate resistance at 60 cycles) through a synchronous contactor which connects the condenser in circuit for exactly one-half cycle and then measures the charge communicated to the condenser by connecting it to a transformer and pulse-stretching vacuum tube voltmeter. The half-cycle contacting period is so phased with respect to the ac power supply that any component of voltage out of phase with that supplied by the bucking potentiometers is discriminated against by a factor of 1000: 1 as compared to 1:1 for the in-phase component. With this setup it has been found possible to detect one microvolt of in-phase voltage in the presence of one millivolt of out-of-phase component. This enables one to balance the computer to .01%. In practice the balance is rarely carried further than 1% since the accuracy of coils to be constructed will certainly be no better. (After a coil has been built to give 1% results, the field is measured and a small correction coil constructed to give the small deviation field.) In order to avoid overloading the amplifier, the whole computer level is controlled by a variac which is gradually turned up as the balancing proceeds. Incidentally the mechanical modulator gives excellent discrimination against all frequencies other than 60 cycles (Q of 100 easily attained) except for odd harmonics in-phase components which are reduced in amplitude by the reciprocal of the order of the harmonic. These, however, are ordinarily not troublesome and in special instances may be descriminated against in the power supply.

The reader has probably noticed that nothing has been said about the accuracy of the approximation of the field at the intermediate points. Little has been done on this subject since the research was of a practical nature and of immediate interest, but I should hazard a guess that the maximum deviation in the field is several factors of ten lower than the magnitude of the field for every 50% change in field. The desired field variation, of course, must be sufficiently small to appear slow to the successive pickup coils in order that a good be obtained. Of even more pressing interest, however, is the behavior of the family of solutions with more variables than constants and in particular the problem of determining the solution in the family with the least resistance. In this field, too, little has been accomplished.

GENERAL DESIGN PROCEDURE

We may now set forth general rules for the experimental determination of field ampere-turns to produce a desired field. Wind inducing coils of reasonable size and form. Wind pickup coils to be placed in critical positions in the field. Use the circuit of Fig. 1 to add the output voltages. Cyclic adjustment of the inducing currents will now produce a null. The inducing currents are now measured and recorded to give directly the required ampere-turns.

This method will, for instance, provide uniform fields for magnetrons with a solenoid length about equal to the length of the anode instead of ten times as long. Likewise for B-H meters and beta-ray spectrometers. Mass spectrometers are particularly susceptible to this type of coils design. Here we may use pancake coils as in the betatron. This should result in a great saving in power over the conventional sphere coils.

At Case Institute of Technology we are now building a 30 Mev air core betatron for continuous operation and a 140 Mev air core machine for pulsed work. These, as well as small coils, cloud chambers, and the like, are being constructed on the basis of calculations outlined in this report. The betatrons incorporate certain features to be described in a later paper.

I wish to take this opportunity to thank Dr. E. F. Shrader and his associates for their helpful advice and encouragement.

New Power Sources for Large Betatrons and Synchrotrons

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The construction of large machines of the betatron and synchrotron type has heretofore been an extremely expensive undertaking--- for two reasons: the existence of a saturation value for the magnetization in iron has necessitated the increase in linear dimension of the accelerator in proportion to the desired particle energy (thus the weight and cost increase as the cube of the energy in the betatron), and, second, the energy stored in the magnetic field increases at least as the square of the particle energy (in a design of air core synchrotron by Blewett the energy needed increases only as the first poser of the particle energy, but this design has several disadvantages).

The first problem has been dealt with in the previous paper by the author as well as by several others. The results of that paper enable one to construct a betatron of reasonable size (half-meter diameter, say) for any particle energy, the only limitations being the adequate removal of heat from the coils and the availability of the necessary electrical energy. The field energy needed in an air core betatron is of the same order as that calculated for an iron core machine of the same energy output and beam current (for very high particle energies the magnetic energy may be less in the air core betatron than for the iron core machine since the axial dimension is necessarily increased by the scale-up necessitated by the use of iron). In any case there remains the considerable problem of supplying power to either of these types of betatron. The necessary energy per acceleration cycle (obtained as $\int_V \frac{H^2}{8\pi} dv \times 10^{-7}$ joules) is approximately $2X^2$ joules[4], where is the electron energy in Mev. For a 300 Mev betatron it is seen that 200,000 joules of energy must be supplied to the magnetic field during each acceleration cycle. At reasonable frequencies betatron resonant circuits have Q of about 100 so that the power dissipation at 60 cps would then be [200,000 × .01 × 2π60] or almost one million watts. In order to reduce the power dissipation to reasonable figures the machine might be operated only six times per second. But the entire 200,000 joules of energy must be stored in condensers in any case (about $60,000 worth of condensers, in fact).

It is the purpose of this paper to describe two new mechanical energy storage devices invented by the author which replace condensers in every betatron (and many another) application at perhaps 2% the cost.

The first of these devices is suitable for the production of the highest energy electrons at very low expense; for instance one such device has been constructed for well under $100 suitable for driving a 140 Mev betatron (or with auxiliary oscillator a 300 Mev synchrotron). Plans and calculations for a machine to supply a 1 Bev betatron indicate a cost of less than $500. The repetition

4. Obtained from approximate field integration and present practice with iron-core machines. $[(2 \pm 1)x^2]$

rate is limited mainly by the power drain and the cooling problem.

Briefly the idea is this --- a small flywheel is employed to store the energy by rotating at a fairly high rotational speed (maintaining an adequate factor of safety against bursting, ofcourse). The energy stored in a steel cylindrical disc flywheel is given by $\frac{1}{2}I\omega^2 = \frac{1}{4} m r^2 \omega^2 = \frac{\pi}{4} r^4 \rho h \times 10^{-7} \omega^2$ joules. From the formulae for the bursting speed of a disc we may find that 50,000 joules can be stored in a flywheel 6 in. diam by 1½ in. thick running at 30,000 rpm with a factor of safety of 2 and tensile strength 50,000 psi (mild steel). Likewise a steel disc 2 in. thick by 12 in. diam running 30,000 rpm will store 2 megajoules of energy (condensers to serve the same purpose would fill a large room).

The line of attack is now evident --- some generator must be attached to the flywheel which will deliver the mechanical energy to the electrical load in less than perhaps 0.03 sec (a limit enforced by radiation losses at the higher electron energies--- but not present in the betatron acceleration of protons). This is an average power conversion of more than 1200 kw even in the case of the 140 Mev machine and amounts to better than 60,000 kw in the case of the 1 Bev betatron (peak powers are about ten times these figures). The accompanying average torques on the generator rotor are given in lb-ft by dividing the generated kw by 4 (approx) for 30,000 rpm. It is thus obvious that a very rugged generator is needed which will give a very high peak power into an inductive-resistive load. A generator which fulfills these conditions is obtained by creating a magnetic flux through the rotating flywheel and by thus utilizing the disc as a Faraday generator, obtaining high powers by leading the discharge current through a series field coil connected so as to produce a negative resistance generator for constant rotation of the disc. No iron magnetic structure is used except for the disc itself. The Faraday disc (or homopolar generator) is ideal also since it has no armature reaction on the magnetizing field. The optimum load resistance is ordinarily fairly low (perhaps .0001 to .01 ohm) since the ohmic resistance of the series field coils increases as the square of the number of turns for a given winding space while the negative resistance of the disc itself increases only directly as the number of turns. It should be mentioned that cooling copper to the temperature of liquid air reduces its resistance by a factor of perhaps six, making possible the preservation of the negative resistance characteristic into a load of perhaps .03 ohm.

In the machine which has been constructed here (Fig.3) power was supplied to the flywheel by an air jet impinging on buckets milled into the edge of the disc. At present the compressed "air" is obtained from a tank of compressed nitrogen for trial runs, but an air compressor will be employed in the final installation. In a very high energy device, however, it would be better to have the air turbine separate from the generating disc, since the latter is subject to brush wear. In my opinion air is preferable to direct drive by mechanical or electrical means because of its convenience (a 100 hp air turbine need be only 6 in. diam and the compressor may be placed in the next room), quietness (after the siren effect has been suppressed), lack of wear, and cooling effect on the generating parts.

The electromechanical forces in such a high-current generator are very large --- of the order of a ton for even a small machine. The largest of these effects is the reaction torque on the stationary conductors exerted by the disc in decelerating. This may amount to 30 ton-ft in the case of the 1 Bev generator. Fortunately it is very easy to get rid of this torque --- one simply uses two discs close together running in opposite directions, thus cancelling all torques and incidentally doubling the output voltage. There need be no forces on the series field coil except that tending to straighten it from its circular form. In the single disc generator there need be no force on the disc (except the decelerating torque) but in the two disc machine there is a repulsion between the two discs which amounts to only a few hundred pounds even in the large generator.

We shall now discuss the electrical equations of such a generator and betatron load. Let n be the number of turns in the series field coil and ω the angular velocity of the disc. Then if r is the disc radius the voltage generated for one ampere-turn in the field coil is given by $\int_0^r H r' \omega \, dr' \times 10^{-8} v$ where $H = H(r)$ which works out to approximately $2 r \omega \times 10^{-8}$ volts/amp-turn-disc. Thus in the two disc generator 30 cm diam the generated voltage is 0.0018 volt/amp turn. For an equivalent single turn field coil("equivalent" because the field coil is actually constructed of many wires beginning and ending at different places in order to permit the use of multiple brushes with balanced forces) the negative resistance of the generator is 1.8 milliohm, and the generator will then work into a total positive circuit resistance of this value or less. An exciting field of about 3000 amp-turns (corresponding to 5 volts generated) is readily obtained by connecting a 10 turn coil of #10 wire to a 6 volt storage battery by means of a relay.

If $-\mathcal{R}$ is the negative circuit resistance and R is the total positive circuit resistance including that of the generator and field coil, and if E is the initial build-up emf then the differential equation of the circuit (during the time the disc remains at sensibly constant speed) is given by $E = [R - \mathcal{R}]i + L\frac{di}{dt}$ where L is the total circuit inductance including that of the generator. The solution of this equation is

$$i = \frac{E}{\mathcal{R} - R}\left[e^{\frac{\mathcal{R}-R}{L}t} - 1\right]$$

and for $\mathcal{R} \sim .0018\Omega$, $R \sim .001\Omega$, $L = 10^{-5}h$, $E = 5v$; $i = 62{,}000\left[e^{80t} - 1\right]$ $\tau_0 = 0.027$ sec

In order to get high rates of energy transfer we decrease the inductance of the betatron until the generator and lead inductance become comparable to that of the betatron. It is easy to see that decrease of inductance increases the efficiency of the generator slightly since the energy dissipated from the closing of the circuit until

is $W = \int_0^{\tau_0} i^2 R\,dt \cong E^2 \int_0^{t_0} \frac{R}{(\mathcal{R}-R)^2} e^{2t\frac{\mathcal{R}-R}{L}}\,dt = \left[W_{magnetic}\frac{R}{\mathcal{R}-R}\right]$

The neglected -1 in $[e^{\frac{R-R_c}{L}}-1]$ increases the losses with increase in L. Thus the inductance should be as low as possible consistent with storing most of the energy in the magnetic field of the betatron itself. Thus for $R \sim .001$, $\mathcal{R} \sim .0036$, $L = 10^{-5}h$ then $\frac{W}{W_{mag}} = \frac{1}{3}$ and the time necessary to reach $W_{mag} = 2 \times 10^6$ joule, $i_0 = 6.3 \times 10^5$ amp with $E = 50V$ is $t_0 = .015$ sec except perhaps for a lag introduced by eddy currents in the generating disc.

It is now of interest to calculate the rate of increase of electron energy at the beginning of the acceleration period (to determine beam current) and near the end of the acceleration (in order to compare this with the energy loss by radiation). Assuming the circuit constants last mentioned we see that the initial rate of change of current is simply expressed as $\frac{E}{L} = 5 \times 10^6$ amp/sec. Since 6.3×10^5 amp corresponds to billion volt particle energies, this initial rate of change of current represents a rate of change of flux which is easily calculated: From the betatron equation $V_{volts} = 300 H_{gauss} r_{cm}$, $H = \frac{V}{300r}$, $\phi = 2\pi r^2 H = \frac{2\pi r V}{300}$ and the induced voltage per turn is given by $v = \frac{d\phi}{dt} \times 10^{-8}$ volts.
$\phi = \frac{i}{i_0}\phi_0 = \frac{i}{i_0}\frac{2\pi r V_{max}}{300} = \frac{i}{i_0}\frac{2\pi r\, 10^9}{300}$ and initially we see $v = \frac{d\phi}{dt}10^{-8} = \frac{7.12\pi r 10^9 10^{-8}}{300}$ or ~ 700 volt, a reasonable value (if slightly lower than desirable). At the end of the acceleration $\frac{di}{dt} = i_0\frac{\mathcal{R}-R}{L} \sim 280\, i_0$ so that $v = 280\frac{2\pi r\, 10^9 10^{-8}}{300} \cong 2800$ volt/turn, a factor of ten higher than that obtained in the 100 Mev GE betatron. This latter figure represents the energy gained by the particle in one revolution and, in order that the betatron shall accelerate the particle, must be greater than the energy lost by radiation[5]. It is well known that heavy particles also may be accelerated in betatron or synchrotron type accelerators, with high efficiency at output energies greater than several times their rest energy --- thus for protons, greater than about 2 Bev. By

5. Given by $880\, E^4/r$ ev/turn; E in 10^9 ev and r in cm.

proper choice of circuit constants and operating conditions the generator-betatron combination should yield electrons with energy greater than 0.7 billion eV. Further increase in the particle energy (limited by radiation loss in the straight betatron) is available by using the generator simply to supply the guide field for a synchrotron in which rf fields up to 50,000 volts/turn may be used. The arrangement introduces not inconsiderable complexity but it decreases the energy requirements by a factor of approximately 4. Still another solution --- for particle energies above a few billion volts protons might be employed (from which the radiation is negligible).

The preceding calculations were worked out on the basis that the disc remain at constant angular velocity. Obviously, since the supply of energy is the rotational energy of the disc, this cannot be so, and in cases where little more than the required energy can be stored the behavior of the machine should be investigated either by approximation or experimant (the complete differential circuit equation appears to be of the fifth order).

It is a surprising and most fortunate fact that the efficiency of energy conversion is about the same with a low $\frac{L}{R}$ betatron ratio as with a high one and that the applicability to betatron acceleration is greatly increased by decreasing this ratio. It should again be mentioned that the forces between the conductors in the betatron and generator are of the order of many tons, and appropriate supporting structures must be used.

A MECHANICAL DEVICE TO REPLACE THE RESONANT CONDENSER IN SMALL AND LARGE AIR-OR IRON-CORE BETATRONS AND SYNCHROTRONS FOR CYCLICAL WORK

In the design of a mechanical contrivance to replace the condensers usually used in a resonant circuit two problems again present themselves: (1) storing the energy supplied to the magnetic field each cycle and (2) converting the stored energy to electromagnetic energy in the inductance of the betatron or synchrotron. The objection to condensers is that they are large and expensive as well as inefficient.

One cannot use simply an alternator to supply the betatron since the machine would have to be wound to generate the total reactive power, which is more than 100 times the real power dissipated. The difficulty is, of course, the large inductance of the betatron. In order to produce an alternating current in this inductance it is necessary to apply many times the voltage required to produce the same current in the betatron resistance. It was here that the idea of supplying current to the betatron instead of voltage appeared (the voltage obviously will have to be developed somehow, and the only excuse for such a line of thought is the valuable results obtained). If a high inductance, of so-far unknown type were used it might be possible to produce the required current and voltages by varying the inductance mechanically with a residual direct current flowing. And this is found to be the case. The generator is actually a variometer (using laminated iron in the low power machines to decrease the losses. even though air gaps of the order of 1 cm thick are present) with equal rotor and stator inductances in series as tightly coupled as possible. The variómeter is hooked across the betatron terminals, and direct current is supplied to the series circuit thus formed by a low resistance dc generator.

The general theory is this: let L_b be the inductance of the betatron, L_1 the inductance of rotor and stator windings individually, and M the mutual inductance of the rotor and stator (highly variable with angle of rotation θ). If i is the current in the circuit at any time t then the torque on the rotor is given by $T = i^2 \frac{\partial M}{\partial \theta}$ (see any textbook on electromagnetics). Now L (total circuit inductance) equals $L_1 + L_1 + 2M + L_b$ so that $\frac{\partial M}{\partial \theta} = \frac{1}{2}\frac{\partial L}{\partial \theta}$ and the mechanical energy put into the system is $W_m = -\int_{\theta_1}^{\theta_2} T\,d\theta$. If for the moment we neglect the circuit resistance, then $i = i(L)$ and $W_m = -\frac{1}{2}\int i^2 dL$. The differentiated energy equation is $\frac{d}{dL}\left[\frac{1}{2} L i^2\right] = -\frac{i^2}{2}$ since $\frac{d}{dL}\int^L \frac{i^2}{2} dL = \frac{i^2}{2}$

$$\frac{i^2}{2} + Li\frac{di}{dL} = -\frac{i^2}{2}, \quad Li\,di = -i^2 dL, \quad i = i_0 \frac{L_0}{L}$$

now $\frac{L_0}{L}$ may easily be made 30, then $i = 30 i_0$ and the mechanical energy becomes 29 times the magnetic energy present originally. This energy is returned to the flywheel on the next half-cycle (minus the losses caused by resistance). In a betatron, resistance may be neglected since less than 2% of the stored energy is dissipated per cycle.

The variometer must be run fast enough so that the rotational energy of the rotor is greater than that demanded by the betatron per cycle, but this is automatically taken care of since the rotor coils are wound for support of an iron core which for all practical designs is sufficiently massive to store the desired amount of energy. The betatron inductance should be such that the required peak currents be about 1000 amperes (this permits winding the variometer with semi-self-supporting copper bar, yet keeps the brush to slipring current reasonable).

An explanation of the above equations may be in order. If there is initially present in the circuit a direct current of magnitude i_0 when the inductance is maximum, the equations indicated that this current will increase to $\frac{i_0 L_0}{L}$ when the flywheel has revolved 180°. The magnetic energy in the betatron has then increased from $\frac{1}{2}L_0 i_0^2$ to $\frac{1}{2}\frac{L_0}{L}L_0 i_0^2$ (a factor of perhaps 30). This increase in field accelerates the electrons. It is to ones advantage to remove this large current from the circuit as quickly as possible, since the useful part of the cycle has passed and only losses are being incurred. This end is attained by special placement of coils on the variometer to make the inductance low for only 1/10 the rotation. Since the magnetic energy is reconverted to rotational energy the cycle is completed with the variometer in position of maximum inductance. A plot of current against time in a typical setup is exhibited in Fig . The dotted axis is the zero of current when the dc excitation is supplied in parallel with the variometer and betatron. With this connection the current in the betatron passes through zero, thus permitting injection of electrons at reasonable energies.

This method of supplying energy has several advantages over the use of condensers. Not the least of these are compactness (2 cubic ft for 150 Mev betatron), ruggedness (as sturdy as a motor), and low cost. Too, in a resonant circuit the average power loss is half the peak power loss, Using the variometer the minimum power loss is .001 that at peak current. In addition, for more than 3/4. of the cycle the current is less than about $\frac{1}{6}$ peak and the mean power is thus less than $\frac{1}{16}$ the peak power, resulting in a real power saving of 85% over the use of condensers. The variometer may be run at high speeds with no increase in losses and an increase in betatron current output (as well as in energy storage capabilities).

Designs have been worked out for several size betatron (including the effect of resistance) but are fairly lengthy and will not be included here (interested persons should consult with the author).

The two electromagnetic machines described should extend the range of available particle energies by a factor of perhaps 50, to about 6 Bev, and may also be used for heavy particles in the higher energy ranges. Their application will permit the construction of large machines at reasonable expense.

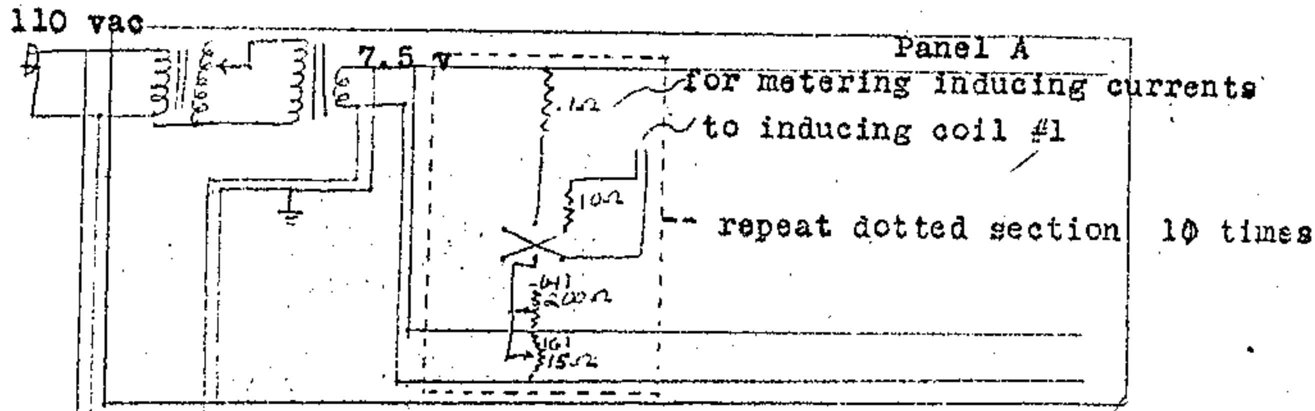

Figure 1a. Inducing coil power supply

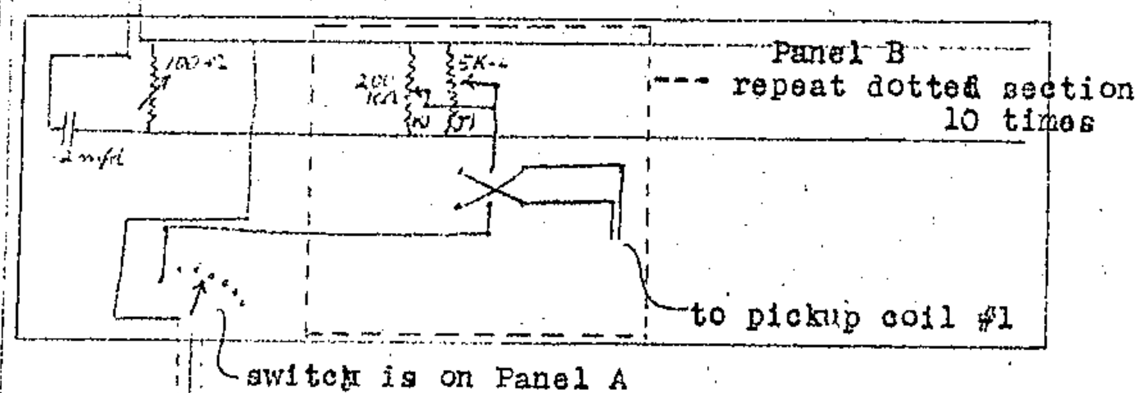

Figure 1b Pickup coil network

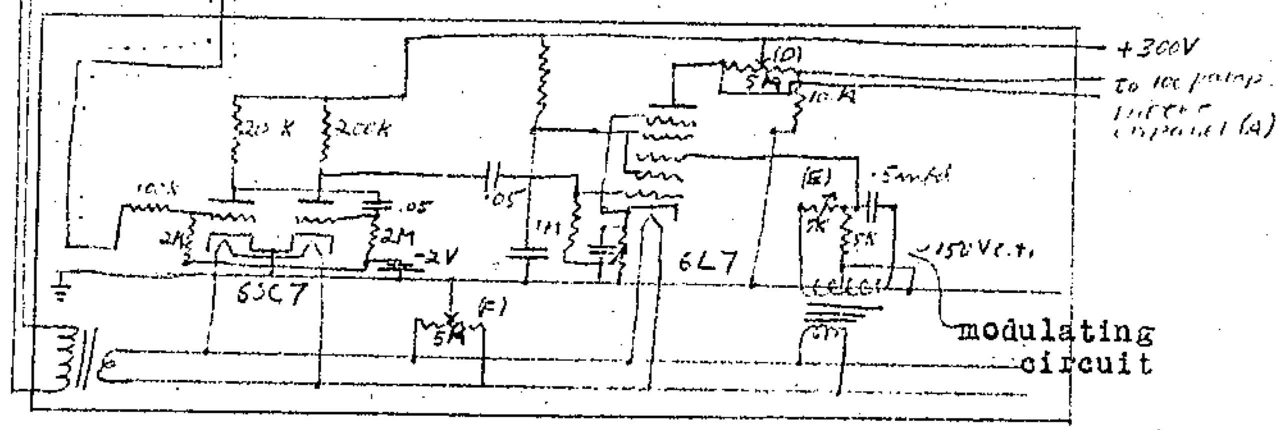

Figure 1c Phase-discriminating amplifier

(Pot F balances out heater hum to less than $.2 \times 10^{-6}$ v., pot E varies the phase of the modulating voltage to compensate for amplifier phase shift--- is set so that no output is obtained for 1 mv input out-of- phase. Pot D is zero set (dc) . Dc component of 6L7 plate current is proportional only to in-phase component obviously. Pots J and K, G and H are coarse and fine adjustments on pickup and inducing voltages respectively. Conveniences such as jacks and jumpers for potentiometer setting of pots and reading of currents have been omitted for the sake of clarity.)

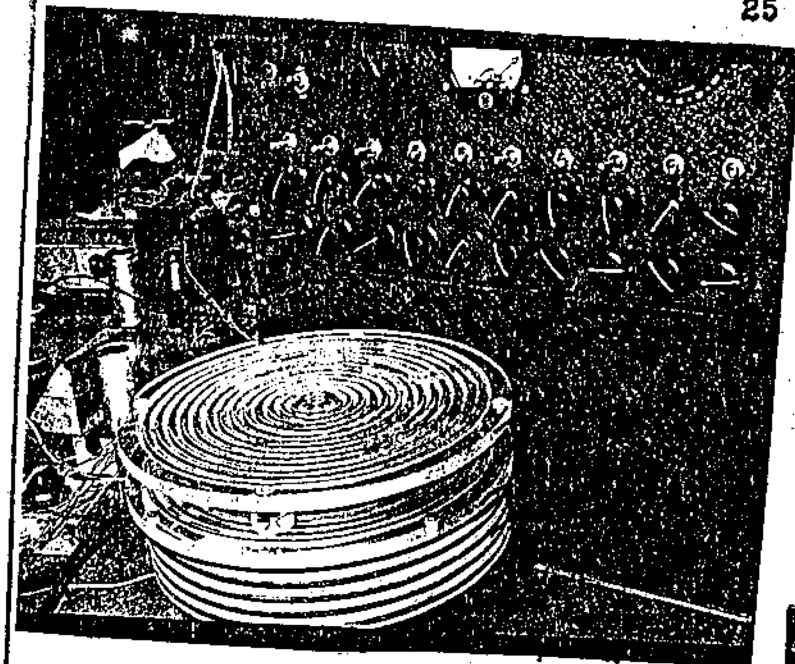

Figure 2

This plate shows inducing coils of two different forms B and C with pickup coils A for betatron-, synchrotron-, mass spectrometer-like fields. Affaratus at left is power supply and circuits not yet mounted on relay panels. D is one of the twenty-six-wire cables to the supply and calculating networks

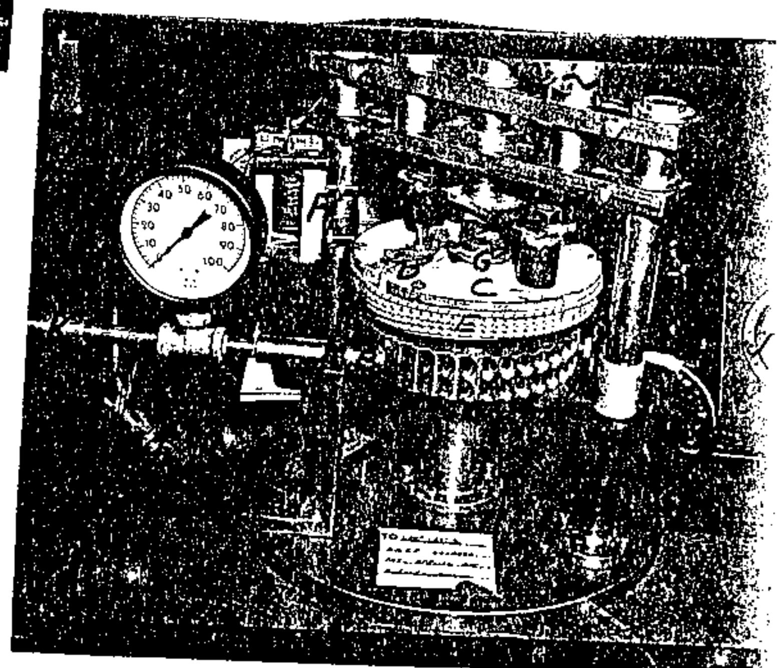

Figure 3

Depicting the turbine rotor A, air-jet (double-nozzle) B, with coil- and brush-block C carrying exciting coil E fed from storage battery by relay F. The end of the series field coil is visible at D onthe experimental disc generator. Here the current is not taken out but is returned to the center of the disc through a flat metal resistor clamped between the copper block L and the center brush-holder G. In this model the brushes are solid copper backed with stiff (30 lb force) springs and heavy copper braid soldered to the brushes. To prevent friction losses the brush- and coil-block is lifted by means of a 5 ft steel lever, the pivot of which is shown at H. When a current pulse is desired the lever is depressed, energizing relay F and coil C by means of a position swithh, and at the same time forcing the copper brushes into contact with the steel disc A. A capacity probe I to the electronic tachometer J is also shown. The tachometer counts the number of impulses (caused by the varying capacity between the probe and the buckets) and reads directly in thousands of rpm. The heavy operating lever H is blocked as shown to prevent rotation of the coil and brush block --- no torque-balancing is used on this small machine (1 ton-ft). The pipe from the pressure reducer and the nitrogen tank is lettered K.